The ADC Table Geometry Used by MX at APS Sector 17

William Lavender

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Introduction

This document describes the geometry used by the ADC table at APS Sector 17 and explains the variable definitions used by MX to describe this table. The ADC table is a fairly normal diffractometer table design with six motors located at three pivot points. Two of these pivot points form a line that is approximately parallel to the path of the X-ray beam. These are called the *upstream outboard pivot point* and the *downstream outboard pivot point*. The third pivot point is called the *inboard pivot point*. Figure 1 shows the geometry.

The locations of the six motors are indicated by bold face arrows that point in the positive direction for each motor. The upstream outboard pivot point has three motors named X1, Y1, and Z1. The downstream outboard pivot point has two motors named Y2 and Z2 and a slide to allow *x* to move freely. Finally, the inboard pivot point has one motor named Z3 and two slides to allow *x* and *y* to move freely. The zero position for the table is shown as *table zero* in Figure 1 and is located at the point where the line connecting the Z1 motor to the Z2 motor intersects the perpendicular dropped from the position of the Z3 motor.

Geometry Calculations

The goal of the geometry calculations below is to allow the position of the center of rotation in six dimensions (x, y, z, θ_{roll}, θ_{pitch}, and θ_{yaw}) to be specified in terms of the positions of the motors X1, Y1, Y2, Z1, Z2, and Z3. It is expected that the table will be rotated in roll, pitch, and yaw by fairly small angles relative to the nominal zeros of roll, pitch, and yaw. For that reason, we will use the small angle approximation

$$\theta \approx \sin\theta \approx \tan\theta \tag{1}$$

in the calculations below. Please note that only the relative position of the rotation center is important, so we may discard extraneous constant factors in the calculations to come.

The position of the rotation center is expressed in terms of six parameters that describe the details of the table geometry. The relative distances separating the motors are expressed using three parameters m_1, m_2, and m_3. Each of these parameters is the distance between the corresponding pivot point and the *table zero* position. Three other parameters called r_x, r_y, and r_z express the position of the rotation center relative to what we are calling the table zero position. They are defined such that r_x is positive if it is upstream of the table zero, r_y is positive if it is inboard of the table zero, and r_z is positive if it is above the table zero.

X Position

Calculating the position of the rotation center in the *x* direction is trivial since X1 is the only motor that moves in this direction. Thus, we have the result

$$\Delta x = x_1 \tag{2}$$

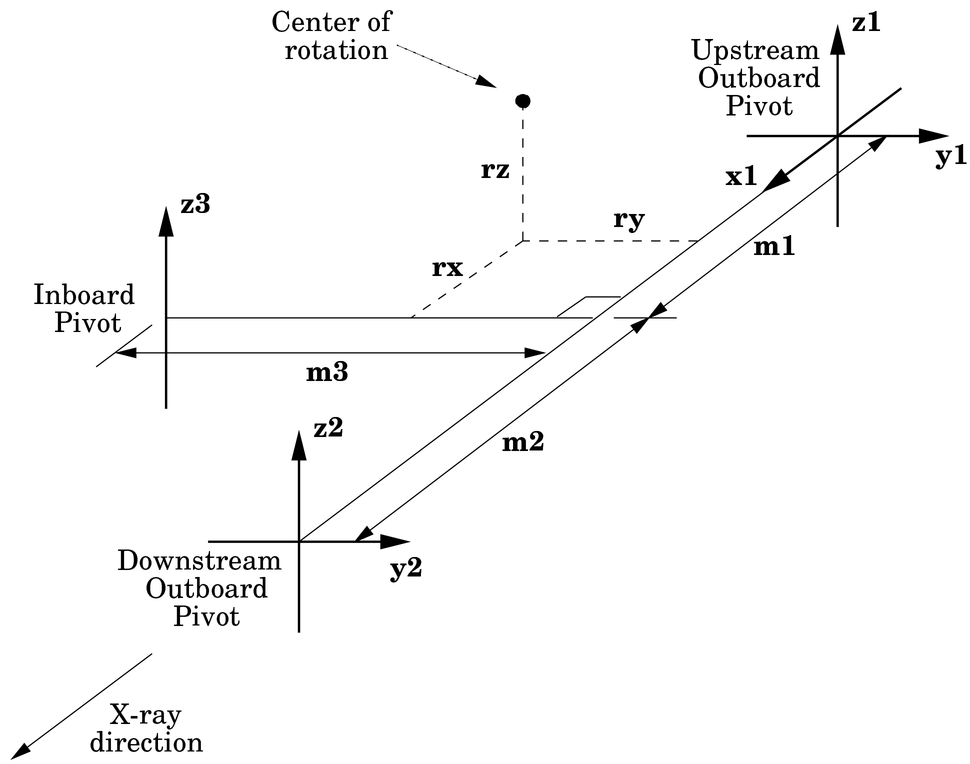

Figure 1: A perspective view of the ADC table.

Y Position

Calculating the y position is only a little bit more complicated than calculating the x position since only the motion of the two motors Y1 and Y2 in a plane need be considered. Their positions projected into the X, Y plane are shown in Figure 2.

By use of similar triangles, we see that

$$\frac{\Delta y - y_2}{m_2 + r_x} = \frac{y_1 - y_2}{m_1 + m_2} \tag{3}$$

which rearranged becomes

$$\Delta y = y_2 + \left(\frac{m_2 + r_x}{m_1 + m_2}\right)(y_1 - y_2) \tag{4}$$

or finally

$$\Delta y = \left(\frac{m_2 + r_x}{m_1 + m_2}\right) y_1 + \left(\frac{m_1 - r_x}{m_1 + m_2}\right) y_2 \tag{5}$$

Z Position

Calculating the change in the z position is significantly more complicated since it depends on three motors (Z1, Z2, and Z3) which move along tracks that are not all in the same plane. For this calculation, we start with the differential expression

$$\Delta z = \frac{\partial z}{\partial z_1}\Delta z_1 + \frac{\partial z}{\partial z_2}\Delta z_2 + \frac{\partial z}{\partial z_3}\Delta z_3 \tag{6}$$

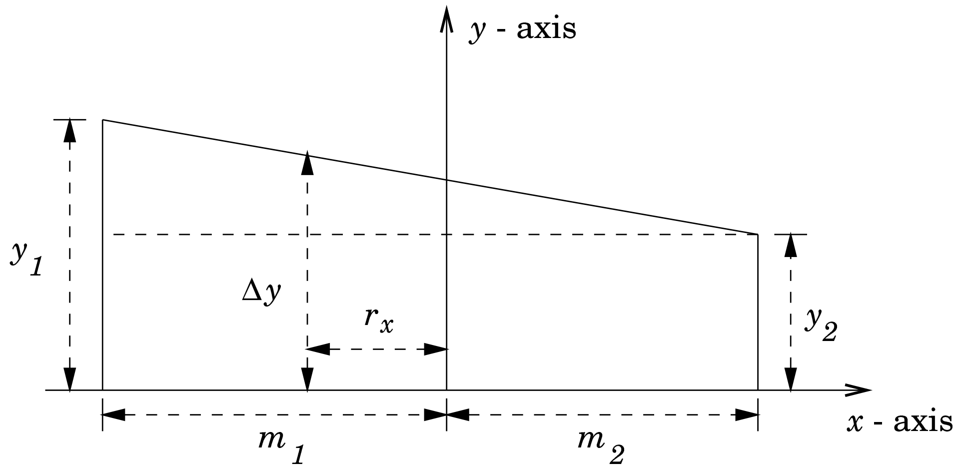

Figure 2: Motion of y as a function of y_1 and y_2.

Using this expression, we can compute the change in height of the rotation center due to each motor separately and then add up the terms afterward. Figure 3 shows the geometry we shall use from directly above the table.

Z3

The easiest term to calculate is the term due to Z3. By similar triangles, we find that the rotation center rises by an amount Δr_z given by the expression

$$\frac{\Delta r_z}{r_y} = \frac{\Delta z_3}{m_3} \tag{7}$$

Rearranged, we get the result

$$\frac{\partial r_z}{\partial z_3} = \frac{r_y}{m_3} \tag{8}$$

Z1

For Z1, since only Z1 is moving, the table rotates on the axis specified by the line from Z2 to Z3. Thus, by similar triangles

$$\frac{\Delta r_z}{u_2} = \frac{\Delta z_1}{m_1 + m_2} \tag{9}$$

so

$$\frac{\partial r_z}{\partial z_1} = \frac{u_2}{m_1 + m_2} \tag{10}$$

u_2 may be found using trigonometry as

$$\tan \alpha_2 = \frac{r_y}{m_2 + r_x - u_2} = \frac{m_3}{m_2} \tag{11}$$

or

$$m_2 + r_x - u_2 = \left(\frac{m_2}{m_3}\right) r_y \tag{12}$$

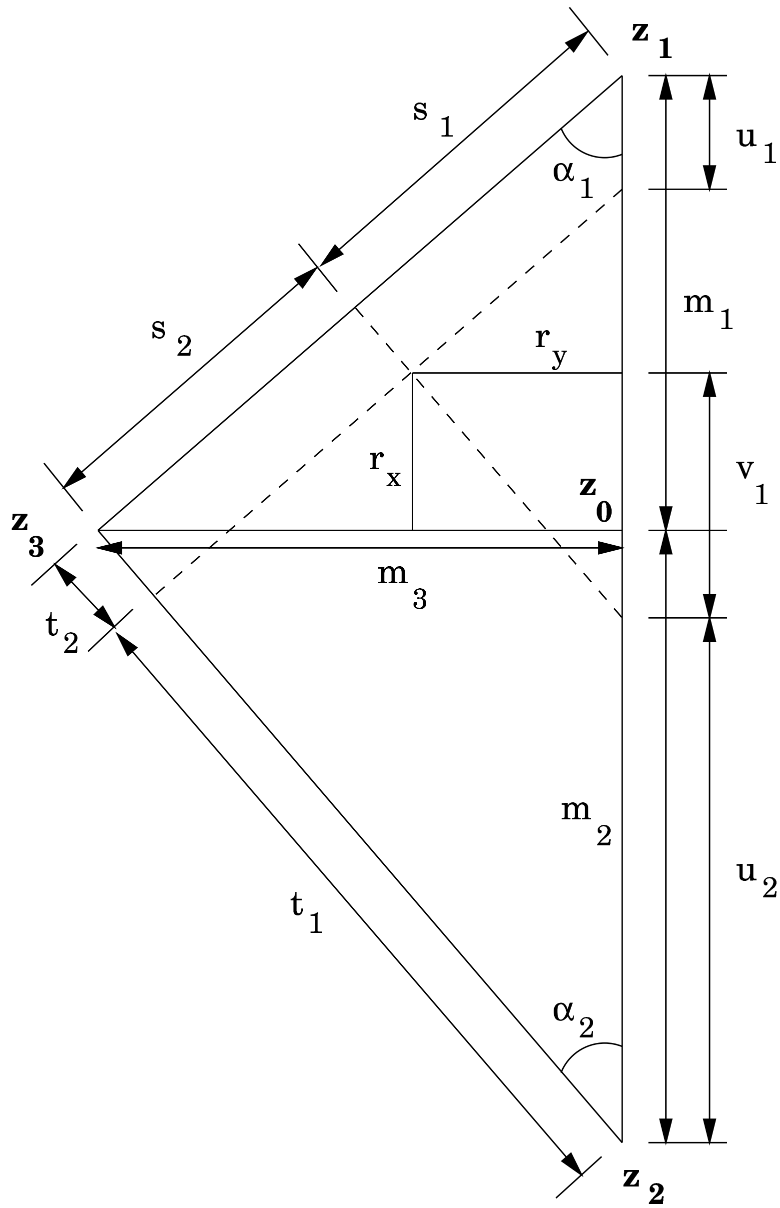

Figure 3: A view of the table from directly above.

which becomes

$$u_2 = m_2 + r_x - \left(\frac{m_2}{m_3}\right) r_y \tag{13}$$

Substituting back in Equation 10, we finally get

$$\frac{\partial r_z}{\partial z_1} = \frac{m_2 + r_x - (\frac{m_2}{m_3}) r_y}{m_1 + m_2} \tag{14}$$

Z2

The calculation for Z2 is similar to that for Z1. Now the table rotates on the axis specified by the line from Z1 to Z3. Thus, by similar triangles

$$\frac{\Delta r_z}{u_1} = \frac{\Delta z_2}{m_1 + m_2} \tag{15}$$

so

$$\frac{\partial r_z}{\partial z_2} = \frac{u_1}{m_1 + m_2} \tag{16}$$

u_1 may be found using trigonometry as

$$\tan \alpha_1 = \frac{r_y}{m_1 - r_x - u_1} = \frac{m_3}{m_1} \tag{17}$$

or

$$m_1 - r_x - u_1 = \left(\frac{m_1}{m_3}\right) r_y \tag{18}$$

which becomes

$$u_1 = m_1 - r_x - \left(\frac{m_1}{m_3}\right) r_y \tag{19}$$

Substituting back in Equation 16, we get

$$\frac{\partial r_z}{\partial z_2} = \frac{m_1 - r_x - (\frac{m_1}{m_3}) r_y}{m_1 + m_2} \tag{20}$$

Z Final Result

Putting together Equations 8, 14, and 20, we get the result

$$\Delta r_z = \left(\frac{m_2 + r_x - (\frac{m_2}{m_3}) r_y}{m_1 + m_2}\right) \Delta z_1 + \left(\frac{m_1 - r_x - (\frac{m_1}{m_3}) r_y}{m_1 + m_2}\right) \Delta z_2 + \left(\frac{r_y}{m_3}\right) \Delta z_3 \tag{21}$$

Yaw

Calculation of the yaw angle is relatively simple since it only depends on the positions of Y1 and Y2. Thus, by inspection we see that

$$\theta_{\text{yaw}} = \frac{y_1 - y_2}{m_1 + m_2} \tag{22}$$

When MX changes the yaw angle using this equation, it may change the y position of the center of rotation. To stop this from happening, the y position of the center of rotation is computed before and after the transformation above. Then, an appropriate Δy value is added to both y_1 and y_2 such that the y position of the center no longer changes.

Roll

The roll angle depends only on the relative heights of positions Z3 and Z0 on Figure 3. Thus,

$$\theta_{\text{roll}} = \frac{z_3 - z_0}{m_3} \tag{23}$$

$z0$, in turn, depends on the heights of Z1 and Z2, via the equation

$$\frac{z_0 - z_1}{m_2 + r_x} = \frac{z_2 - z_1}{m_1 + m_2} \tag{24}$$

or

$$z_0 = z_1 + \left(\frac{m_2 + r_x}{m_1 + m_2}\right)(z_2 - z_1) \tag{25}$$

Similarly to the yaw, a change of the roll angle using this equation may change the z position of the center of rotation. To stop this from happening, the z position of the center of rotation is computed before and after the transformation above. Then, an appropriate Δz value is added to z_1, z_2, and z_3 such that the z position of the center no longer changes.

Pitch

The pitch angle depends on the relative heights of Z1 and Z2, so by inspection

$$\theta_{\text{pitch}} = \frac{z_1 - z_2}{m_1 + m_2} \tag{26}$$

Handling a pitch change using the above equation is somewhat more complicated in that it can change both the roll angle and z position of the center of rotation. Since a change in roll can also change the z position, we first compensate for the change in roll and then afterward compensate for the change in z.